

## TEMPERATURE DEPENDENT LEVEL DENSITY

Takehiro Matsuse

Faculty of Textile Science and Technology,  
Shinshu University, Nagano 386, Japan

Sang-Moo Lee

Institute of Physics and Tandem Accelerator Center,  
University of Tsukuba, Ibaraki 305, Japan

**Abstract:** We propose a new empirical formula of temperature dependent level density parameter "a" in which the shell structure energy is evaluated by using the nuclear ground state- and Liquid Drop Model-binding energy. The fission mass distributions populated through heavy ion reactions for the medium mass systems are well accounted for in terms of the Extended Hauser-Feshbach Method using the temperature dependent level density.

(Temperature dependent level density, Shell structure energy, Fission mass distribution, Extended Hauser-Feshbach Method, Liquid Drop Model, Scission-Point Model)

Introduction

The fission mass distribution is one of the most important characteristics of the nuclear fission process. Although a large number of experimental data on the mass distribution in fission for various nuclei under a variety of conditions including heavy-ion induced fission has been available up to now, no suitable theory yet exists which explains all the observations<sup>1)</sup>.

The scission-point model<sup>2)</sup> incorporating with deformed-shell effects can provide successfully an interpretation of a difference in threshold energies for symmetric and asymmetric mass splits in the fission of radium and actinum isotopes as well as the other physical quantities. It is, however, quite unsatisfactory in calculating the widths of fission mass distributions; though the peak positions are well reproduced the calculated widths are always narrower than the observed ones.

The same feature was also seen in heavy-ion induced fission reactions on the medium mass systems ( $A_P + A_T \leq 100$ ); the peak positions of mass distributions are well accounted for in terms of the Extended Hauser-Feshbach Method<sup>3)</sup> but not their widths properly.

The aim of this paper is to propose a temperature dependent level density and to present its effects in broadening fission mass distributions.

Formula of Temperature Dependent Level Density

Using the simple and illustrative model<sup>4)</sup> developed by Bohr and Mottelson for thermodynamical properties of atomic nuclei, an empirical formula of level density parameter "a", depending on the nuclear temperature, is proposed in which the shell structure energy is evaluated by using the observed- and the Liquid Drop Model-binding energy.

Then the shell structure energy  $E_{sh}(\tau)$  at finite temperature and the entropy  $S$  can be approximately expressed in terms of the shell structure energy of a ground state  $E_{sh}(0)$  as follows,

$$E_{sh}(\tau) \simeq E_{sh}(0) \tau^2 \frac{\cosh(\tau)}{\sinh^2(\tau)}$$

$$S \simeq \tilde{S} \left[ 1 + E_{sh}(0) \left( \frac{2\pi}{\hbar\omega_{sh}} \right)^2 \frac{3(\tau \coth(\tau) - 1)}{g_0 \tau \sinh(\tau)} \right]$$

where  $\tau$  is measured by the nuclear temperature  $T$  as  $\tau = 2\pi^2 T / \hbar\omega_{sh}$ ;  $\hbar\omega_{sh}$  is a shell spacing.

These formulas mean that if the reasonable ground state shell structure energy  $E_{sh}(0)$  is evaluated we can easily estimate a temperature dependence of  $E_{sh}(\tau)$  and  $S(\tau)$ .

To evaluate the  $E_{sh}(0)$  we use the observed binding energy  $B_{A,Z}^{(obs)}$  from Ref. 5 and the calculated binding energy  $B_{A,Z}^{(cal)}$  of the Liquid Drop Model developed by Seeger and Perisho<sup>6)</sup> for spherical nucleus

without shell and pairing correction;

$$E_{sh}(0) = E_{sh}^{(obs)} = B_{A,Z}^{(obs)} - B_{A,Z}^{(cal)} + \Delta$$

where  $\Delta$  is a pairing energy. The smooth part of  $S$  is determined as

$$\tilde{S} = 2\tilde{a}T$$

where the observed average value of  $\tilde{a} = A/8$  is used.  $U$  and its temperature  $T$  are related as follows<sup>4)</sup>,

$$U = \tilde{a}T^2 + E_{sh}(\tau) - E_{sh}(0)$$

where  $U$  is the excitation energy with pairing correction;  $U = E - \Delta$ .

In addition the level density parameter " $a$ " is connected with the excitation energy  $U$  and entropy  $S$  as,

$$S = 2\sqrt{aU}$$

To calculate " $a$ " in this manner, we use the following values for the remained parameters;  $g_0 = \frac{3}{2}\epsilon_F A$  with  $\epsilon_F = 30$  MeV,  $\hbar\omega_{sh} = 41/A^{1/2}$  and  $\Delta = 12/A^{1/2}$  MeV.

Fig. 1 shows the calculated level density parameters " $a$ " for the nuclei appeared in Ref. 7). The solid line shows the smooth part of the level density parameter  $\tilde{a} = A/8$ . The bulk trend of the fluctuation of level density parameter from system to system is well reproduced. In Fig. 2 the excitation energy dependence of " $a$ " is shown for the nuclei along the  $\beta$ -stable line. As can be seen the effect of shell structure on the level density parameters decreases gradually with increasing the excitation energy and remains approximately up to 50 MeV for the most of nuclei and even up to 100 MeV for the magic shell region.

#### Effects of Temperature Dependent Level Density (TDL) in fission mass distributions

Although fission-like phenomena have been observed in heavy-ion induced reactions for the medium mass systems ( $A_P + A_T \leq 100$ ) and the various interpretations have also been made, the present situation is still puzzling<sup>3)</sup>.

The main reason why the reaction mechanism cannot be identified unambiguously is due to the fact that the observed mass distributions change quite rapidly from symmetric- to asymmetric-distribution depending on the system<sup>8)</sup>.

We apply the Extended Hauser-Feshbach Method to calculate mass distributions for the fission-like process in the medium mass systems.

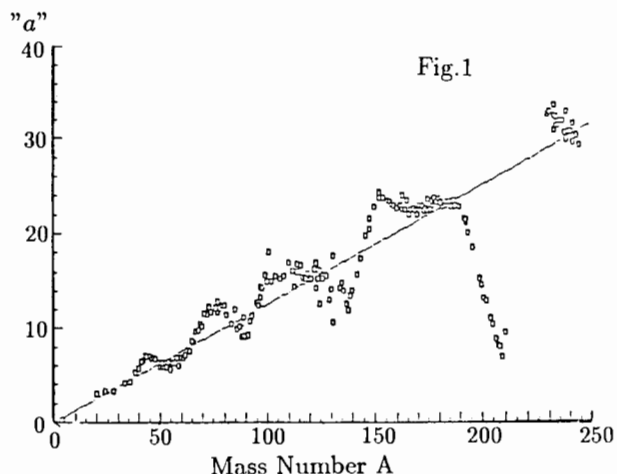


Fig.1 The calculated level density parameter " $a$ ." The solid line shows  $\tilde{a} = A/8$ .

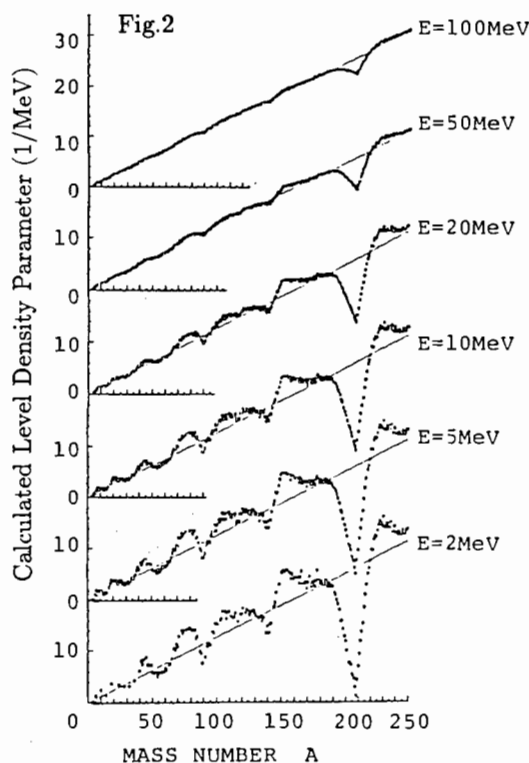


Fig.2 The excitation energy dependence of " $a$ ". The solid line shows  $\tilde{a} = A/8$ .

In the present calculation we take into account the following open channels; 1) particle evaporation  $n$ ,  $p$  and  $\alpha$ , 2)  $\gamma$ -ray and 3) binary fission.

The number of phase space for evaporation and  $\gamma$ -decay is calculated as usual. In contrast to the Transitional State Model we use the method proposed by Ericson<sup>9)</sup> to evaluate the number of phase space for fission as follows,

$$N_{L_\alpha, I_\alpha}^{(J)}(E_\alpha) = \int \int \sum_{(I_{\alpha_1}, I_{\alpha_2}) I_\alpha} \rho(I_{\alpha_1}, \epsilon_{\alpha_1}) \rho(I_{\alpha_2}, \epsilon_{\alpha_2}) T_{L_\alpha}(E_\alpha) \delta(\epsilon_{\alpha_1} + \epsilon_{\alpha_2} + E_\alpha + Q_\alpha - E^*) d\epsilon_{\alpha_1} d\epsilon_{\alpha_2}$$

where  $E^*$ ,  $\epsilon_{\alpha_1}$ ,  $\epsilon_{\alpha_2}$ ,  $E_\alpha$  and  $Q_\alpha$  are the excitation energy of compound nucleus, that of the fission fragment 1, that of the fragment 2, the relative kinetic energy and the fission Q-value, respectively.  $I_\alpha$  denotes a spin of each fragment.

The physical parameters involved in the calculation are essentially the same with Ref. 3) except for the Q-value calculation. The binding energy of ground state is used to calculate Q-values.

In the present work the level density formula given in Ref. 10) is adopted;

$$\rho(I, E) = \frac{1}{24} \left( \frac{a\hbar^2}{2\mathcal{J}} \right)^{\frac{3}{2}} (2I+1) a \frac{e^{2\sqrt{X}}}{X^2}$$

where  $X = a[E - \frac{\hbar^2}{2\mathcal{J}} I(I+1) - \Delta] = aU$  and  $E$  is the excitation energy of a fission fragment.

In Fig. 3 and 4 the dashed lines indicate the calculated results for  $^{32}\text{S} + ^{76}\text{Ge}$  and  $^{32}\text{S} + ^{59}\text{Co}$  systems where the usual level density parameters  $\bar{a} = A/8$  for all nuclei are used. As seen in these figures, the  $^{32}\text{S} + ^{76}\text{Ge}$  system shows the symmetric mass distribution but the  $^{32}\text{S} + ^{59}\text{Co}$  the asymmetric one. This feature can be understood as being due to the effect of Q-values; the exit channel of half mass fragments has the deepest Q-value for the  $^{32}\text{S} + ^{76}\text{Ge}$  system but the exit channel of mass division with projectile and target has the deepest one for the  $^{32}\text{S} + ^{59}\text{Co}$  system. Though the positions of peak value for the mass distributions are well reproduced in this way, the widths calculated are too narrow compared to the experimental results. Therefore we introduce the TDLD which makes the width of fission mass distributions broaden.

The solid lines in Fig. 3 and 4 show the calculated results with TDLD giving better agreements with the experimental mass distributions than the dashed lines.

Figure 5 shows the final results of calculation using TDLD for  $^{32}\text{S}$ -induced fission on various targets together with the experimental data<sup>8)</sup>. The agreement between theory and experiment is rather satisfactory in both the widths and the symmetric and asymmetric behaviour of mass distributions.

The effect of TDLD in broadening the fission mass distribution can be understood as follows,

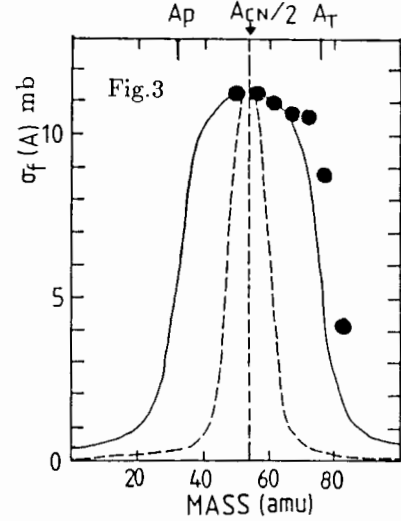


Fig.3 The Fission Mass Distribution for  $^{32}\text{S} + ^{76}\text{Ge}$  at 198 MeV<sup>11)</sup>. The solid line is calculated with the TDLD and the dashed line without the TDLD.

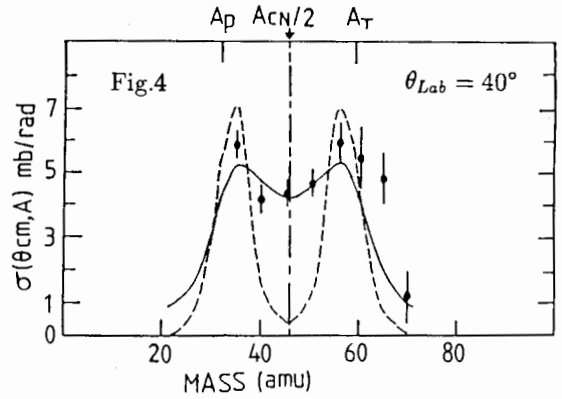


Fig.4 The fission Mass Distribution for  $^{32}\text{S} + ^{59}\text{Co}$  at 198 MeV<sup>12)</sup>. The symbols are the same as indicated in Fig.3.

- 1) The channel with a deeper Q-value gives a larger fission cross section  $\sigma_f$  and  $E_{sh}^{(0)} \leq 0$ . On the contrary the channel with a shallower Q-value gives a small  $\sigma_f$  and  $E_{sh}^{(0)} \geq 0$ .
- 2) When  $E_{sh}^{(0)} \leq 0$ , "a" becomes small and consequently  $\sigma_f$  becomes also small. When  $E_{sh}^{(0)} \geq 0$ , "a" becomes large and  $\sigma_f$  becomes also large. Combining two effects the width of fission mass distribution becomes wide.

### Conclusion

We propose an empirical formula of level density parameter "a", depending on the nuclear temperature, in which the shell structure energy is evaluated by using the observed- and Liquid Drop Model-binding energy.

The effect of shell structure on the level density parameters decreases gradually with increasing the excitation energy and remains approximately up to 50 MeV for the most of nuclei.

The fission mass distributions formed through heavy ion reactions for the medium mass systems are well accounted for in terms of the Extended Hauser-Feshbach Method using the present Temperature Dependent Level Density.

We make grateful acknowledgement for Mrs. Keiko Yuasa-Nakagawa's typing the manuscript.

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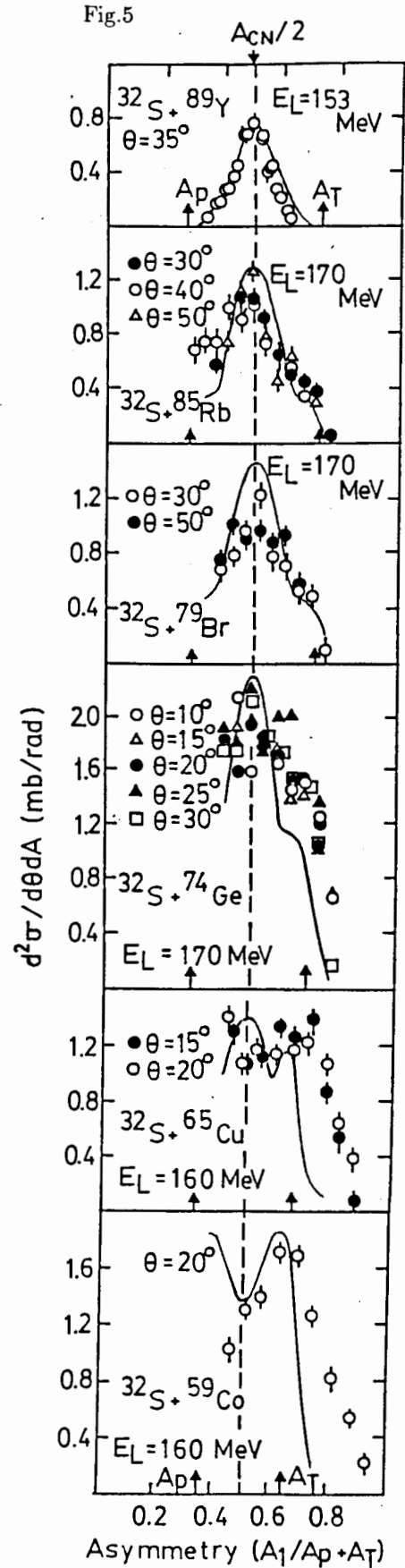


Fig.5 Mass distributions of  $^{32}\text{S}$ -induced fission-like process<sup>8)</sup>. The solid lines are the results of present calculation with TDLD.